An Alternative Approach to 1/2-Integer Resonant Extraction Using a Supplementary 0th-Harmonic Quadrupole Circuit

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Leaping clean over all the fine formalism of Mr. W.R. Hamilton†, we take it as a given that:

With a 0th harmonic octupole driving term the separatrices are 2 circles, with their overlap defining the stable phase space region.

$$\left[x \pm \left(\frac{q_2\beta}{6\lambda}\right)^{1/2} \sin\left(\frac{\psi}{2}\right)\right]^2 + \left[x' \mp \left(\frac{q_2\beta}{6\lambda}\right)^{1/2} \cos\left(\frac{\psi}{2}\right)\right]^2 = \left(\frac{\Delta\beta}{6\lambda}\right)$$

q_2 = 1/2-integer driving term
λ= 0th harmonic octupole field
Δ is the fractional tune separation from the ½ integer
ψ determines the phase space orientation at the septum

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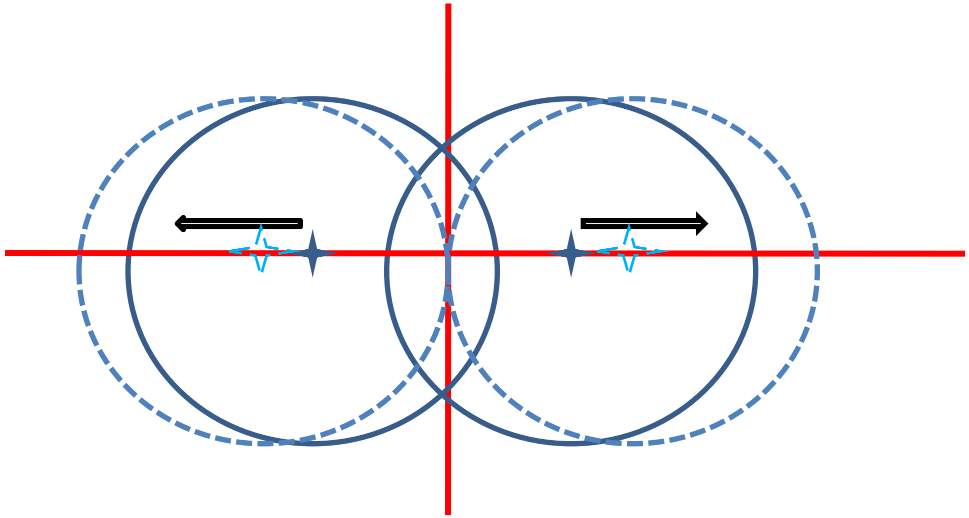
† For an explanation of these terms & where they all come from you could start by looking at:

"A Simplified Analysis of Resonant Extraction from the Main Injector",
John A. Johnstone, MI note MI-0091,

or;

For a much more complete & elegant mathematical treatment, consult Leo's tome:

"Preliminaries Toward Studying Resonant Extraction from the Debuncher (DRAFT),
Leo Michelotti & John Johnstone, Mu2e-doc-556.



$$\left[x \pm \left(\frac{q_2\beta}{6\lambda}\right)^{1/2} \sin\left(\frac{\psi}{2}\right)\right]^2 + \left[x' \mp \left(\frac{q_2\beta}{6\lambda}\right)^{1/2} \cos\left(\frac{\psi}{2}\right)\right]^2 = \left(\frac{\Delta\beta}{6\lambda}\right)$$

Assuming a fixed circle radius the q_2 driving term is systematically increased to pull the circles apart, thereby decreasing the stable area to zero.

This approach to resonant extraction was used exclusively in the Main Ring, again in the Tevatron, and still today in the Main Injector.

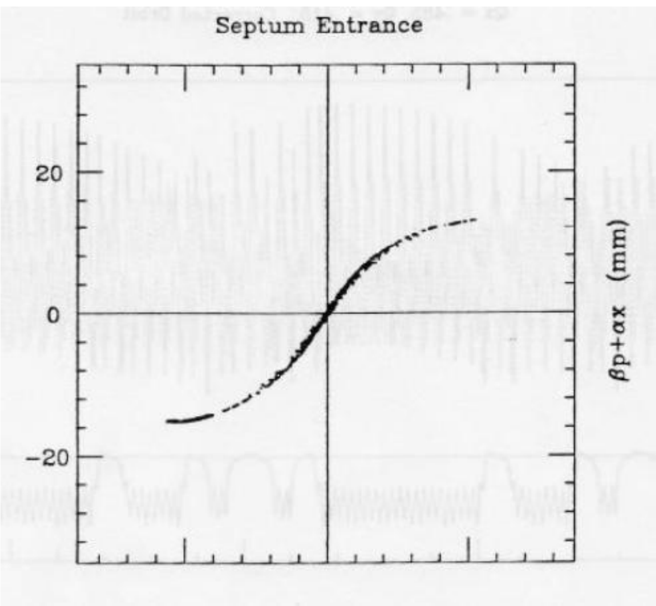
Generally the results have been pretty good……

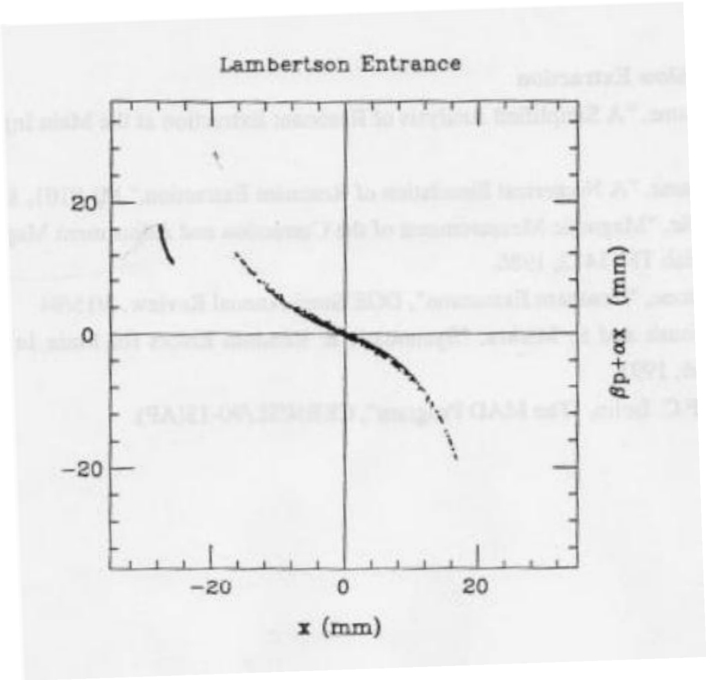
Resonant Extraction from the MI @ 120 GeV/c:

$Cx = +5$, & $\Delta p_{95}/p = \pm 0.04\%$:

Tune Spread $\Delta_{95} = 0.015 \pm 0.002$

$\Rightarrow$ ~12 mm *separation at Lambertson*



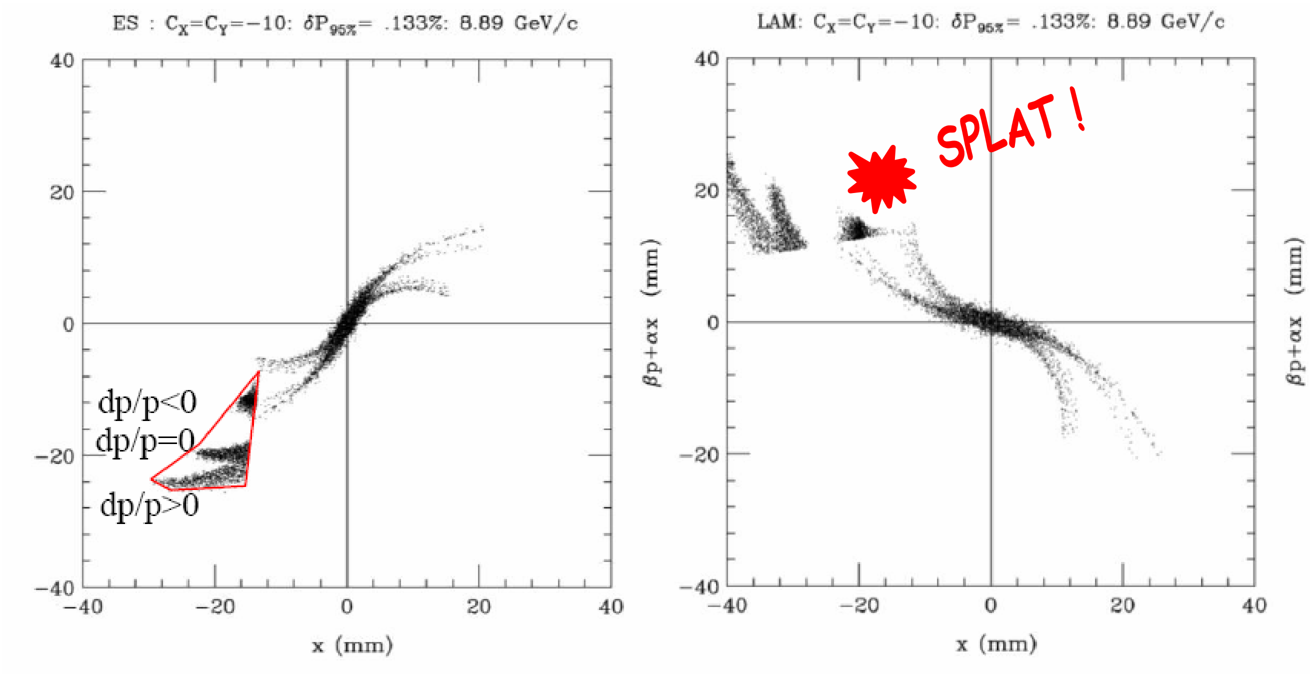


But ! ….

Simulated Extraction from the Recycler (actually MI) @ 8.9 GeV/c:
Cx = -10, & $\Delta p_{95}/p = \pm 0.133\%$:

Tune Spread $\Delta_{95} = 0.025 \pm 0.0133$

$\Rightarrow$ ***ZERO*** *separation at Lambertson !*



So what is going wrong !? …

$$\left[x \pm \left(\frac{q_2\beta}{6\lambda}\right)^{1/2} \sin(\psi/2)\right]^2 + \left[x' \mp \left(\frac{q_2\beta}{6\lambda}\right)^{1/2} \cos(\psi/2)\right]^2 = \left(\frac{\Delta\beta}{6\lambda}\right)$$

The pitfall in this technique, in which only the q_2 circuit is ramped, lies in the assumption that the separatrices' radii do not vary appreciably, or, in other words, that the "base" or "bare" tune is a valid parameter to characterize the beam.

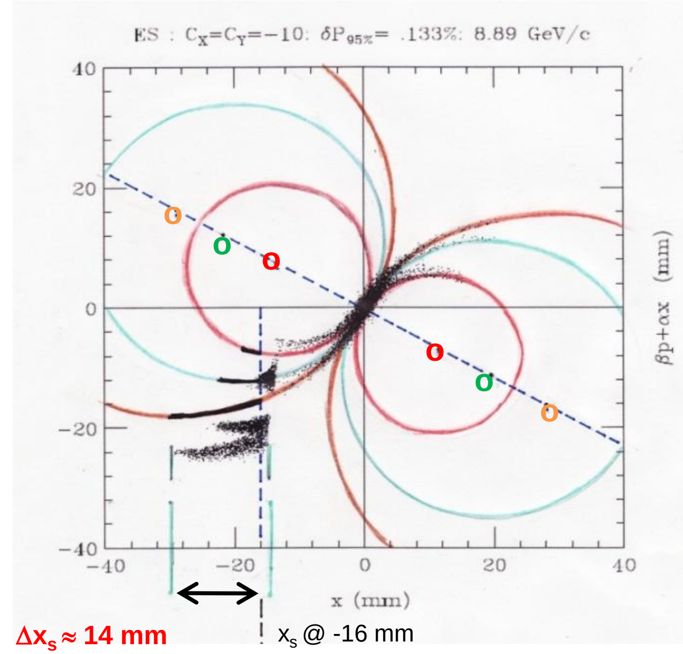
In the RR example, though, the tune was a strong function of momentum:

$$\Delta(p) = \Delta_0 - C_x \cdot \Delta p / p_0$$

Δ_0 = 0.025, Cx = -10, & $\Delta p_{98}/p_0$=±0.155%, giving a range of tune separations:

$$\boxed{\Delta(p) = 0.0095 \Rightarrow 0.0405}$$

leading to variation in the radii by a factor of ~2 or more between high & low momentum particles.



ε_{95} = 32.5 π μm
β_{sep} = 36.0 m
λ = 235 m^{-1}
$\chi/2 \approx$ 65°

		radius (mm)	q_2 (x 10-2)
Δ	= 0.0095	15.6	0.15 → 0.95
	0.0250	25.3	1.61 → 2.50
	0.0450	32.3	2.47 → 4.05

Try something different — add a q_0 tune control circuit...

- Conventional 1/2 integer extraction is an octupole-dominated approach, which assumes that the intrinsic tune spread is small, & relies on a strong non-linear field to produce amplitude-dependent tune shifts that lead to unstable particle orbits.
- It has been demonstrated that this technique is totally inappropriate (and conceivably, impossible) for beams with "large" tune spreads (not necessarily due to chromatic effects).
- It is worth exploring a quadrupole-dominated extraction scheme, in which the difficulties associated with the tune spread might be alleviated — or perhaps even used to advantage.

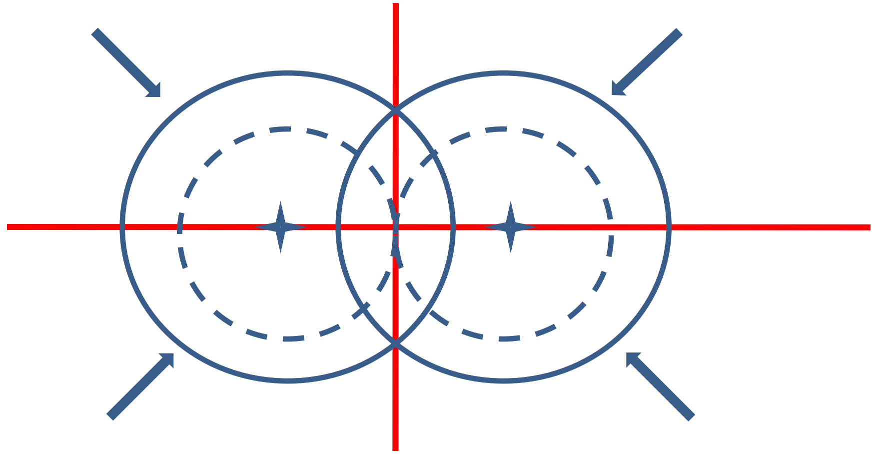
A quadrupole-dominated scheme can be implemented by augmenting the half-integer q_2 extraction quads with a 0^{th} harmonic circuit q_0 to control the tune separation $\Delta(p)$.

$$\left[x \pm \left(\frac{q_2\beta}{6\lambda}\right)^{1/2} \sin(\psi/2)\right]^2 + \left[x' \mp \left(\frac{q_2\beta}{6\lambda}\right)^{1/2} \cos(\psi/2)\right]^2 = \left(\frac{\Delta\beta}{6\lambda}\right)$$

In contrast to the scheme discussed previously, the intention now is to keep the separatrices' centers fixed while decreasing the circle radii to extract particles until zero stable phase space area is obtained.

The selection of the circle centers' locations, through q_2/λ & $\chi/2$ is, of course, one component in the whole question of optimization. The radii can be controlled, though, via the 0^{th} harmonic q_0 quadrupoles;

$$\Delta(p) = \Delta_0 - C_x \cdot \frac{\Delta p}{p_0} - q_0$$



Supposing that the q_2 & λ values have been selected according to some appropriate criteria, then the area of the stable phase space for a particular emittance ε can be expressed in terms of the q_2/Δ ratio as:

$$\boxed{\pi\varepsilon\cdot\frac{\beta}{2}=\left(\frac{q_2\beta}{6\lambda}\right)\cdot\left(\frac{\Delta}{q_2}\right)\cdot\left\{\sin^{-1}\left[\sqrt{1-\frac{q_2}{\Delta}}\right]-\sqrt{\frac{q_2}{\Delta}}\cdot\sqrt{1-\frac{q_2}{\Delta}}\right\}} \quad *$$

- q_2 and λ are constants.
- $\left(\frac{q_2\beta}{6\lambda}\right)^{1/2}$ is the distance from the phase-space origin to the separatrices' circle centers, and is constant by design.

- The ratio q_2/Δ that initiates extraction of the $\pi\varepsilon$ area can be solved numerically from equation *:†

  @ $\frac{q_2}{\Delta}=\frac{q_2}{\Delta}\Big|_{initial}$ $\pi\varepsilon$ is marginally (un)stable.

- At the end of extraction:

  @ $\frac{q_2}{\Delta}=1\Rightarrow\Delta\equiv q_2$ $\pi\varepsilon$ = 0.

† Although the $\pi\varepsilon$ equation might appear formidable, numerical solution is quite simple & fast.

- The 2 final results presented for Δ on the preceding slide are the foundation upon which implementation of this 1/2-integer extraction scheme depends. Given fixed values of q_2, λ (and, therefore, fixed circle centers), and emittance ε, the conclusions reached are:

$\therefore \Delta = \Delta|_{initial} \rightarrow q_2$ covers the entire course of ε extraction.

The q_2/Δ solutions to ✱ are unique — they are simply numbers, and no explicit dependence on the chromatic tune shift (or any other source of tune spread) appears in these Δ boundary conditions.

The Δ limits, therefore, must be valid for any & all $\Delta p/p$.

Returning to ✱, and the definition:

$$\Delta(p) = \Delta_0 - C_x \cdot {\Delta p}/{p_0} - q_0$$

It is clear (or should be) that, for any $\Delta p/p$, q_0 can always be chosen such that Δ will fall within the extraction range.

- In case the plot of this story has not yet been sufficiently flogged, we ***finally*** arrive at the punch line:

$\because \Delta = \Delta_{|initial} \rightarrow \boldsymbol{q}_2$ covers the total extraction of emittance ε, irrespective of $\Delta p/p$:

- variation of the separatrices' circle radii are identical over the course of extraction;
- step-size at the septum is identical for all $\Delta p/p$, and;
- Extracted beam phase-space trajectories are identical for the entire range of $\Delta p/p$.

A simple model illustrating this scheme follows. Goodbye 21st century; Hello, the 1980's! ……

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A MODEL DEMONSTRATION

$\epsilon = 25\pi$ μm $\qquad \Delta = 0.02 \pm 0.01 - g_0$

CHOOSE (FOR NO GOOD REASON):

$$r_c = \left(g_2 \beta / 6\lambda \right)^{1/2} = 20 \text{ mm}$$

$$\beta = 15 \text{m} \qquad X/2 = 0$$

END OF EXTRACTING A PARTICULAR $\Delta p/p$:

$$g_0 = \Delta_0 - C_x \cdot \Delta p/p - g_2 \quad (\text{i.e.: } g_2/\Delta \equiv 1)$$

START OF EXTRACTING A PARTICULAR $\Delta p/p$:

$$g_2/\Delta = 0.80255$$

WITH $g_0 \equiv 0$ AT START OF $\Delta = 0.01$ EXTRACTION

$$\Rightarrow \lambda = 50 \text{ m}^{-1}$$

CIRCLE CENTER @ $x = \pm 20$ mm

VARIATION IN CIRCLE RADIUS

$$r_s = 22.33 \rightarrow 20.00 \text{ mm}$$

OVER THE COURSE OF EXTRACTION

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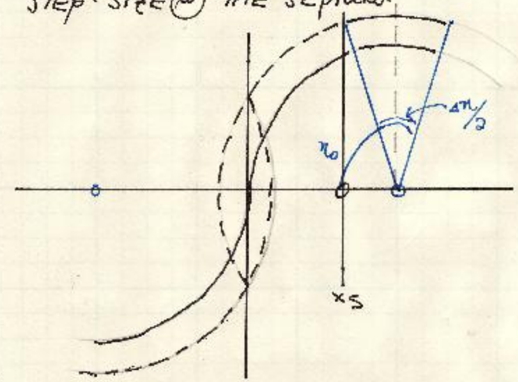
STEP · SIZE @ THE SEPTUM:



IF THE LAMBERTSON IS 90° D/S OF THE SEPTUM BEAM SIZE IS MINIMIZED FOR

$$n_0 = \pi/2 - \Delta n_0/2$$

$$\Delta x_s = \left(\Delta\beta / 6\lambda \right)^{1/2} \cdot 2 \cdot \sin\left(\Delta n / 2 \right)$$

$$= r_s \cdot \frac{8\pi\Delta}{\sqrt{1 + (4\pi\Delta)^2}}$$

$$\boxed{\Delta x_s = 4.0 \rightarrow 5.6 \text{ mm OVER EXTRACTION}}$$

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EXTRACTION INEFFICIENCY

$$f = \left(\frac{6\lambda}{\Delta\beta}\right)^{1/2} \frac{\omega}{8\pi\Delta \cdot \cos\left(\frac{\Delta\nu}{2}\right)\left[1 - \sin\left(\frac{\Delta\nu}{2}\right)\right]}$$

$$x_S = x_o - r_S \sin\left(\frac{\Delta\nu}{2}\right) = r_S\left[1 - \sin\left(\frac{\Delta\nu}{2}\right)\right]$$

$$f \rightarrow \frac{\omega}{8\pi\Delta \cdot x_S \cdot \cos\left(\frac{\Delta\nu}{2}\right)} \simeq \frac{\omega}{8\pi\Delta \cdot x_S}$$

$x_S = 16$ mm

$\omega = 0.1$ mm

$$\boxed{\Rightarrow f \simeq 3\%}$$

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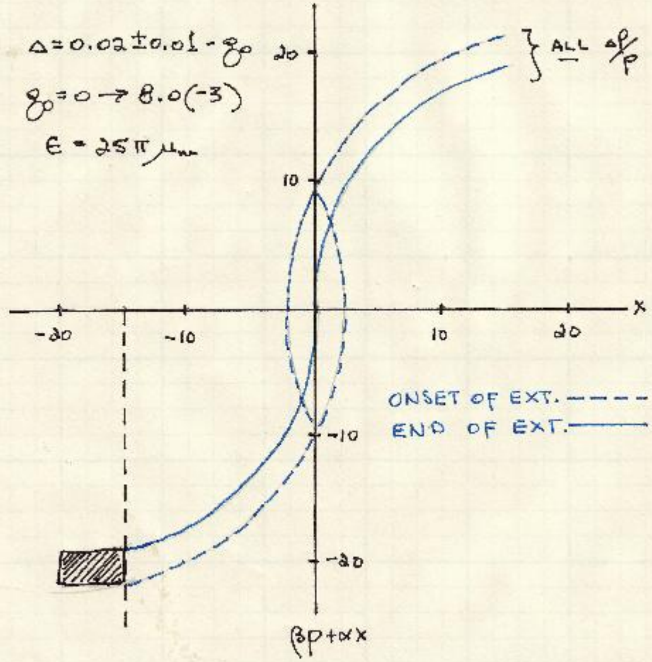

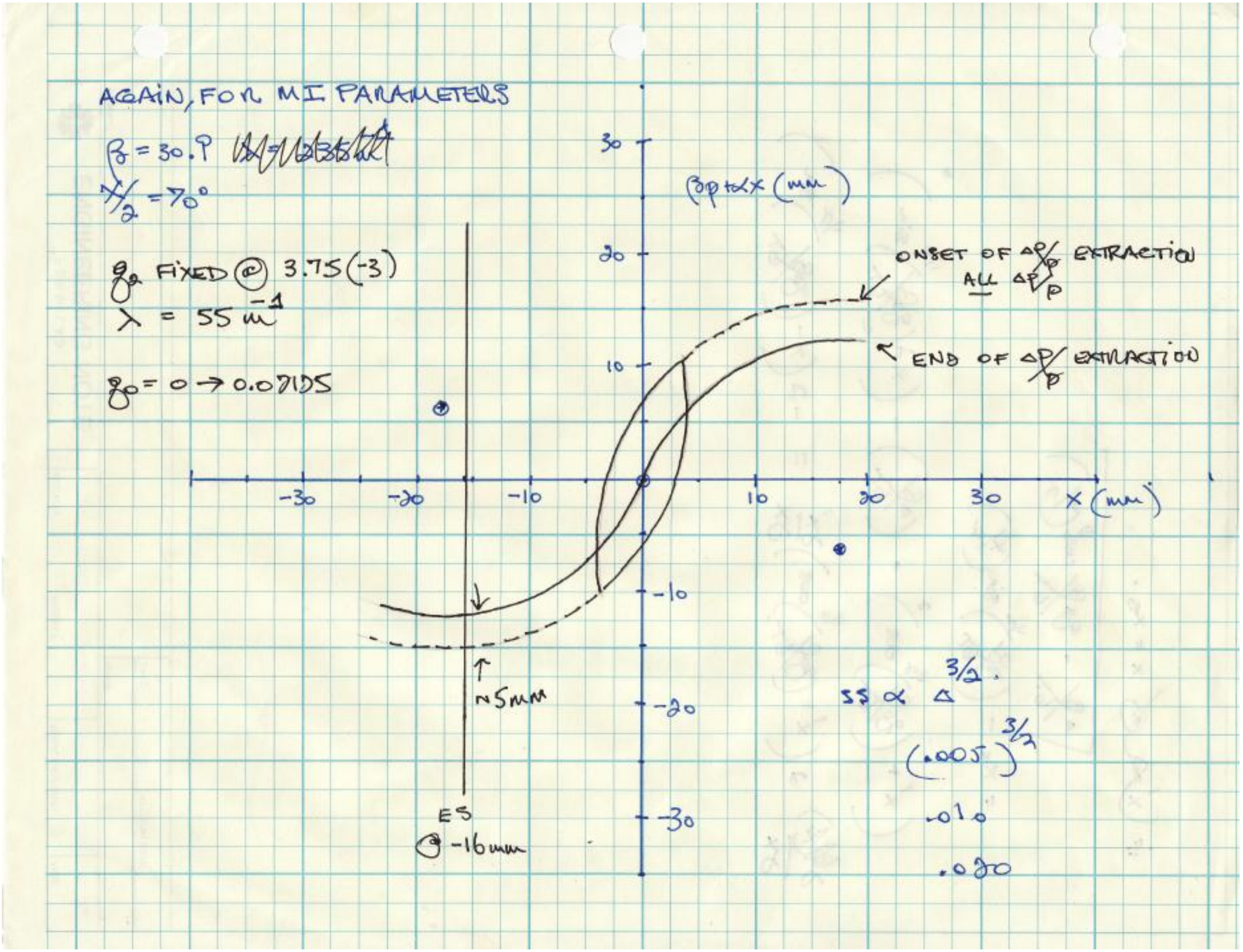
AGAIN, FOR MI PARAMETERS
β = 30.9
X/2 = 70°
g2 FIXED @ 3.75(-3)
λ = 55 m^-1
g0 = 0 → 0.07125
βp to dx (mm)
ONSET OF ΔP/P EXTRACTION
ALL ΔP/P
END OF ΔP/P EXTRACTION
x (mm)
~5mm
ES
@ -16mm
SS ∝ Δ^3/2
(.005)^3/2
.010
.020

Summary

- The addition of a q_0 tune circuit has potential for greatly improving half-integer extraction in the event of a large tune spread.

- Because of the great similarity between the MI and Recycler lattices this technique can be tested in MI – the MI has all the necessary components already installed.

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