Parametric Analysis of Beam Transmission in the Mu2e Extinction Channel

Eric Prebys

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Abstract

Two schemes have been proposed to achieve the extinction required by the Mu2e experiment. Both involve magnets or combinations of magnets in resonant circuits. For this reason, both will have some lateral beam motion during the transmission window, which could potentially result in unwanted beam loss. This note compares the MECO scheme to the scheme presented in the Mu2e Proposal. In addition, two variations of the latter are analyzed, in which a small amount of a high harmonic is added to reduce beam motion during the transmission window.

1 Introduction

The Mu2e experiment relies on an 8 GeV primary proton beam consisting of short (≈100 nsec FW) bunches, separated by 1.7 μsec. It is vital that out-of-bunch beam be suppressed at the level of 10^{-9} or less. Because of the high repetition rates, it is assumed that any solution will involve some combination of resonant magnets, and at least two have been proposed.

In a separate document [1], we presented a generic treatment of the effect of the AC dipole field can be generically treated as a shift in phase space, as shown in Figure 1. If we assume that the beam admittance A is well defined elsewhere in the beam line and is equal to the admittance of the collimator (or target), then beam will be completely extinguished by an angular kick corresponding to twice the full angular amplitude corresponding to the admittance, or

$$\Delta\theta = 2\sqrt{\frac{A}{\beta_x \beta\gamma}} \quad (1)$$

where β_x is the betatron function in the bend plane and β and γ have their usual definitions.

However, because the beam has finite longitudinal and lateral distributions, some of the beam which is nominally "in time" will be extinguished. The transmission efficiency will depend on these distributions and and the details of the lateral beam motion.

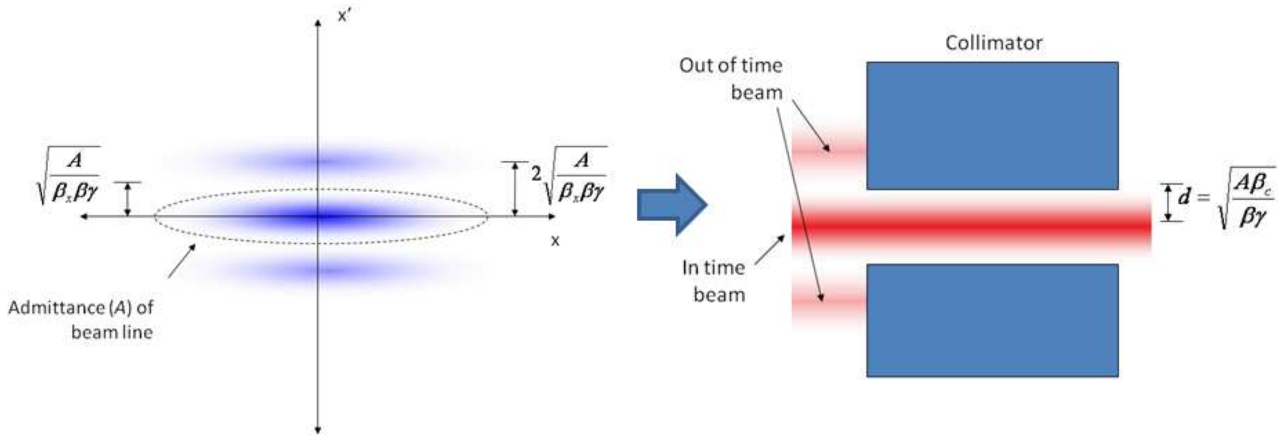

Figure 1: Effect of the AC dipole field in phase space. Beam line admittance A is indicated by the ellipse. Shown at right is effect of the dipole at the collimator (or other defining aperture).

2 Parametric Description of Transmission Aperture

Whether we are discussing beam being extinguished by hitting a collimator or missing the target, we can represent the transmitted distribution by a admittance A associated with a defining half-aperture d

$$A = \sqrt{d\beta_x\beta\gamma} \tag{2}$$

Assuming a symmetric aperture, the fraction of transmitted beam will be

$$\eta(\sigma_x, d, \Delta x) = \frac{1}{\sigma_x\sqrt{2\pi}}\int_{-d}^{d} e^{-\frac{(x-\Delta_x)^2}{2\sigma_x^2}}dx \tag{3}$$

where Δx is the displacement of the centroid of the beam. If we express this displacement as a fraction of the half aperture ($\delta \equiv \Delta x/d$), this becomes

$$\eta(\sigma' \equiv \sigma_x/d, \delta) = \frac{1}{\sigma'\sqrt{2\pi}}\int_{-1}^{1} e^{-\frac{(x'-\delta)^2}{2\sigma'^2}}dx' \tag{4}$$

For a gaussian beam, the transverse distribution is described by

$$\sigma_x = \sqrt{\frac{\beta_x\epsilon}{6\beta\gamma}} \tag{5}$$

where ϵ is the 95% normalized emittance. Thus

$$\frac{\sigma_x}{d} = \sqrt{\frac{\epsilon}{6A}} = \sqrt{\frac{\kappa}{6}} \rightarrow \sigma' = \frac{\sigma_x}{d} = \sqrt{\frac{\kappa}{6}} \tag{6}$$

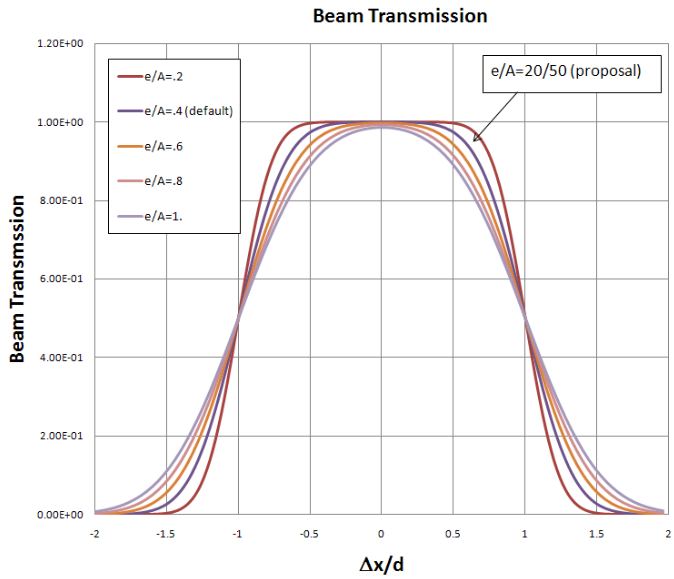

Figure 2: Beam transmission as a function of $\delta \equiv \Delta x/d$ for various values of ϵ_{95}/A.

where $\kappa \equiv \epsilon/A$ is a dimensionless repesentation of the beam size. Plugging this into (4), the transmission effienciency can be expressed in terms of two dimensionless parameters

$$\eta(\kappa, \delta) = \frac{1}{\sqrt{\kappa/6}\sqrt{2\pi}} \int_{-1}^{1} e^{-\frac{(x'-\delta)^2}{2(\kappa/6)}} dx' \tag{7}$$

3 Modeling of Transmission Efficiency

We consider four models for the waveform of the extinction kicker, as illustrated in Fig 3:

- A sine wave running at half the bunch rate, as described in the Mu2e proposal [2].
- The same sine wave mofied by a sine wave with the opposite polarity, 1/17 the amplitude, and 17 times the frequency (5.1 MHz),to reduce the beam slewing to exactly zero at the nominal bunch time.
- The configuration described above, but with the higher harmonic having 2/17 the amplitude of the primary to expand the transmission window somewhat further.
- The configuration proposed for MECO [3], comprised of magnets operating at the first three harmonics of the bunch frequency, with relative amplituces of 1:.74:.63.

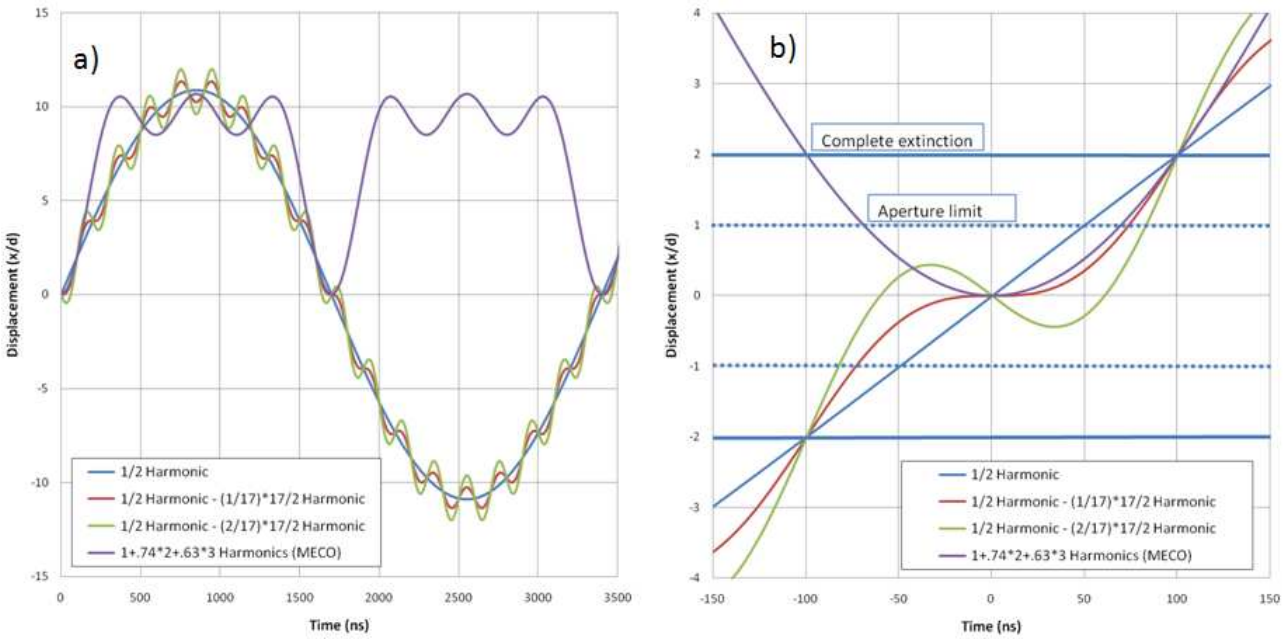

Figure 3: The extinction kicker waveforms which are analyzed in this note. In all cases, amplitudes have been normalized so that complete extinction is achieved at ± 100 ns. Figure a) shows the complete waveform over two bunch cycles, while b) shows the detail near the nominal bunch time. The defining aperture is indicated, as is the amplitude corresponding to complete extinction.

In all cases, the amplitude has been normalized such that the amplitude is twice the aperture (ie, full extinction) at ± 100 ns.

In each case, a limiting admittance of 50 π-mm-mr was assumed and a numerical integration was done to calculate beam transmission for a gaussian beam with a 95% emittance of 5 and 20π-mm-mr. Trasmission efficiency was calculated as a function of the standard deviation of the gaussian time distribution.

4 Results

Results are shown in Figure 4. We see that the single harmonic sine wave presented in the proposal has serious concerns with transmission efficiency except at extremely short bunch lengths. All schemes lose fairly significant beam at the nominal bunch length of $\sigma_t = 38$ ns; however, that was based on putting six booster batches into the Accumulator/Debuncher. Recent schemes, with lower intensity should enable shorter bunches. If the bunches are < 20 ns, then any of the other three schemes should work. In general, there is little sensitivity to the beam emittance; however, the scheme with the larger 4.8 MHz component suffers some beam loss even for short bunches,

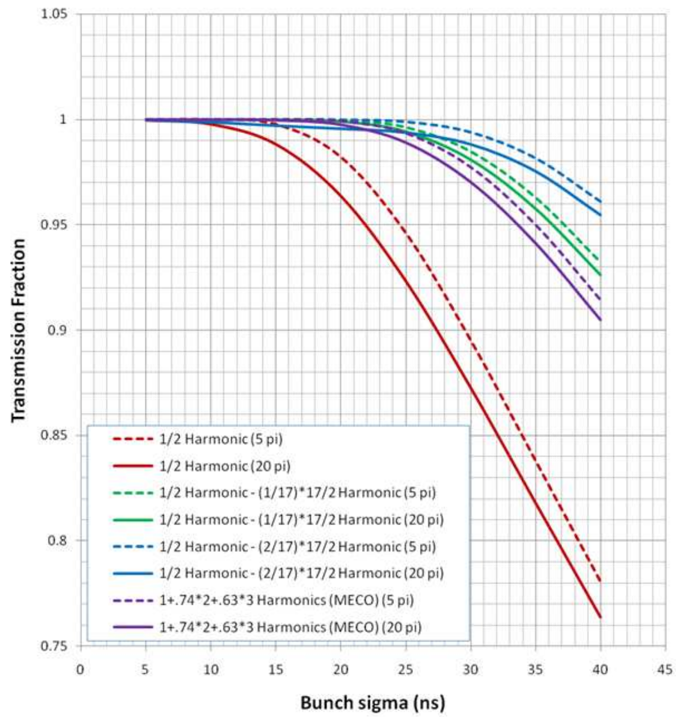

Figure 4: Beam transmission as a function of σ_t for the four exinction kickers. Dashed and solid lines show the results for ϵ_{95} of 5 and 20 π-mm-mr, respectively.

because the beam slewing during the transmission window brings the beam unacceptably close to the limiting aperture.

5 Conclusions

The simple sine wave exctinction scheme described in the proposal result in unacceptable beam loss unless the bunches can be made extremely short (σ_t of about 10 ns or less). The MECO scheme performs significantly better in this regard. A new proposal involving a small amount of a 5.1 MHz harmonic looks very promising. In comparing this to the MECO proposal, consideration will need to be made of the relative difficulty of the higher frequency compared to the elimination of one harmonic and significantly reduced amplitude.

References

[1] E. Prebys, "Optimizing of AC Dipole Parameters for Beam Extinction", http://mu2e-docdb.fnal.gov, Mu2-doc-534-v1

[2] Mu2e Collaboration, "Proposal to Search for $\mu N \rightarrow eN$ with a Single Event Sensitivity Below 10^{-16}", FNAL Proposal E-973, http://mu2e-docdb.fnal.gov, Mu2e-doc-388-v1. Sec. 6.5.

[3] W. Molzon, "Proton Beam Extinction", MECO-EXT-05-002 (2005), http://meco.ps.uci.edu/old/ref_design/MECO-EXT-05-001V1.02.pdf